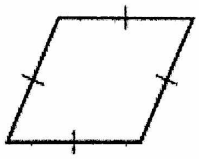
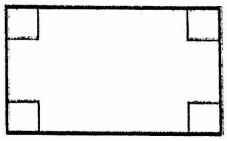
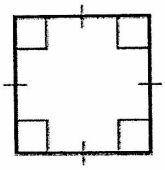
Name: Date: Period: 5

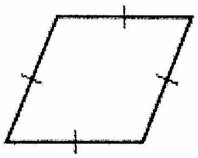
Properties of Special Parallelograms

Lesson Objective SWBAT classify and prove that a quad. is a rhombus, rectangle, ~~or~~ and/or square

DEFINITIONS		
Rhombus	*Rectangle*	*Square*
		
ll-gram w/ 4 ≅ sides	ll-gram w/ 4 ≅ ∠s	ll-gram w/ 4 ≅ sides AND 4 ≅ ∠s.

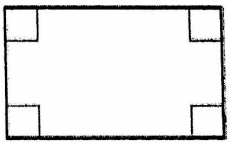
Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.



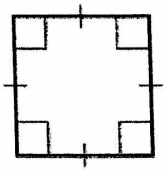
Rectangle Corollary

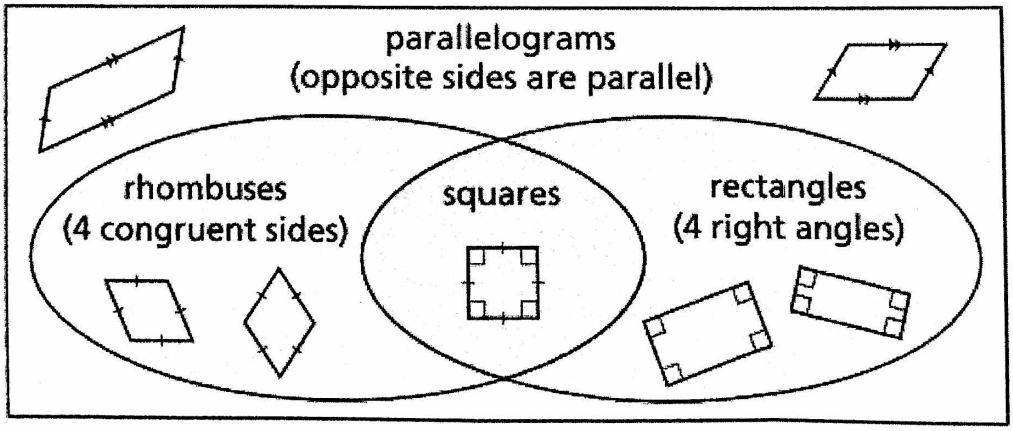
A quadrilateral is a rectangle if and only if it has four right angles. (≅)



Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.



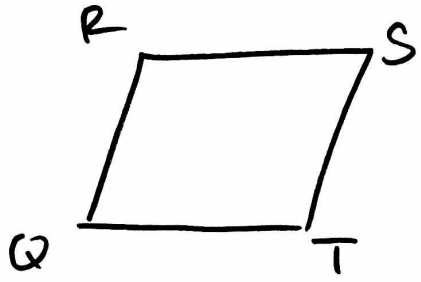


Example

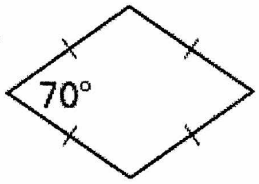
1. For any rhombus QRST, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a diagram
   Remember, the order of the vertices in the name "QRST" tells you the order of how to "connect" the vertices!

a. $\angle Q \cong \angle S$

b. $\angle Q \cong \angle R$



2. Classify the special quadrilateral. Explain your reasoning.

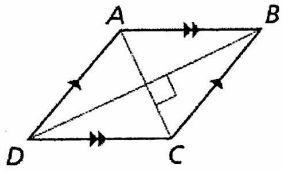


rhombus

quad. w/ 4 ≅ sides

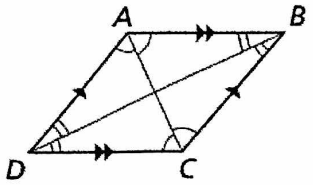
Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.



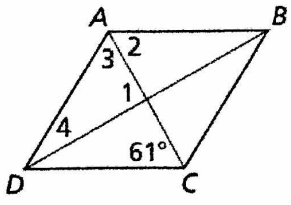
Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.



Example

3. Find the measures of the numbered angles in rhombus ABCD.



$m\angle 1 = 90°$

$m\angle 3 = 61°$

$m\angle 2 = 61°$

$m\angle 4 = 29°$

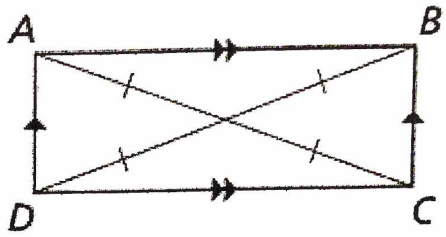
4. In the diagram above, what is $m\angle ADC$? What is $m\angle BCD$?

$m\angle ADC = 58°$

$m\angle BCD = 122°$

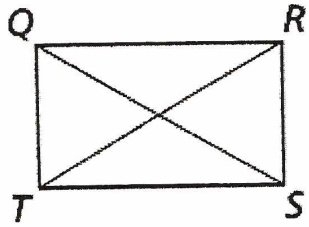
Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.



Example

5. In rectangle QRST, $QS = 5x - 31$ and $RT = 2x + 11$. Find the lengths of the diagonals of QRST.



$5x - 31 = 2x + 11$

$3x = 42$

$x = 14$

$QS = RT = 39$

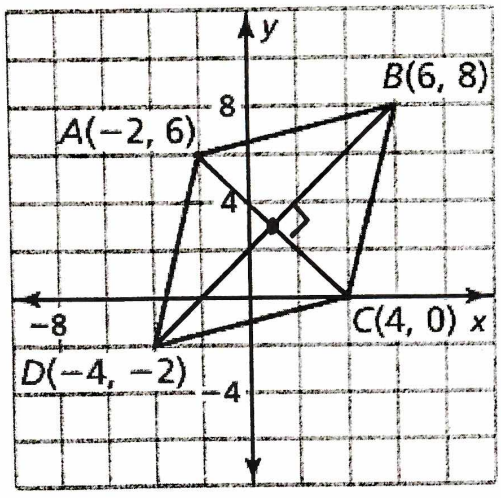
Think About It…What should we show to prove that a parallelogram is a square? *Hint: Use the Square Corollary…*

① prove it is a rhombus

② prove " " " ~~rect~~ rectangle

Example

6. Decide whether quadrilateral ABCD below is a *rectangle*, *rhombus*, or *square*. Give ALL names that apply.



① parallelogram?

mdpt $\overline{AC} = \left(\frac{-2+4}{2}, \frac{6+0}{2}\right) = (1, 3)$

mdpt $\overline{BD} = \left(\frac{-4+6}{2}, \frac{-2+8}{2}\right) = (1, 3)$

mdpt is same, ~~so ll gram~~. for both diagonals, so they bisect each other. So, ABCD is a ll-gram

② rhombus?

slope $\overline{AC} = -1$

slope $\overline{BD} = 1$

slopes ~~are ⊥~~. multiply to -1, so $\overline{AC} \perp \overline{BD}$. diagonals

So, ABCD is a rhombus.

③ rectangle?

$AC = \sqrt{(-2-4)^2 + (6-0)^2}$

$= 6\sqrt{2}$

$BD = \sqrt{(-4-6)^2 + (-2-8)^2}$

$= 10\sqrt{2}$

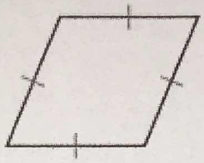
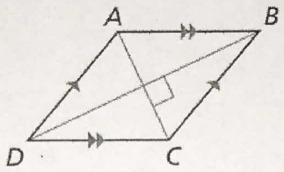
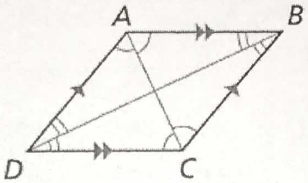
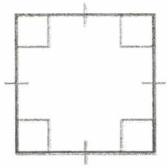
since $AC \neq BD$, then ABCD is not a rectangle.

④ square?

no, since it is not a rectangle.

rhombus

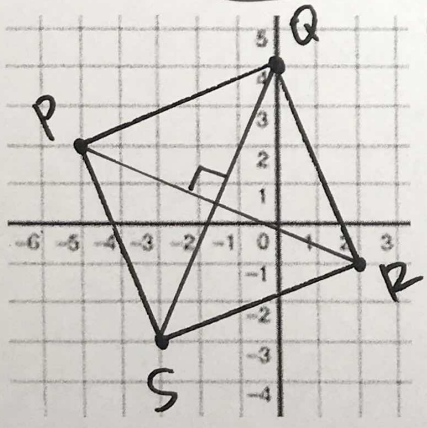
use Diagonals!.

Ways to Prove that a Quadrilateral is a SPECIAL Parallelogram **REMEMBER:** If you are not given it is a ‖-gram, then you must first prove that it is a ‖-gram!			
Shape	*Theorem or Definition*	*Diagram*	*Method for Coordinate Plane*
Rhombus	Show that all four sides are congruent		dist. formula (4x)
	Show that diagonals are perpendicular ★		slopes (2x)
	Show that diagonals bisect the angles		
Rectangle	Show that all four angles are right angles		slopes (4x)
	Show that diagonals are congruent ★		dist. formula (2x)
Square	Show it is a rhombus AND a rectangle!		① method rhombus ① method rectangle

Example

already given, don't have to prove ☺

7. Decide whether parallelogram PQRS is a *rectangle*, *rhombus*, or *square*. Give ALL names that apply.



① rhombus?

slope $\overline{PR} = -\frac{3}{7}$

slope $\overline{QS} = \frac{7}{3}$

Since slopes (of diags) multiply to −1, the diags are ⊥. So, PQRS is a rhombus.

② rectangle?

$PR = \sqrt{(-5-2)^2+(2-(-1))^2} = \sqrt{58}$

$QS = \sqrt{(-3-0)^2+(-3-4)^2} = \sqrt{58}$

square

since the diags are = in length, $\overline{PR} \cong \overline{QS}$. So, PQRS is a rectangle.

③ square? yes, since PQRS is a rhombus AND a rectangle.